1 Fig. 3 shows the curve $y = x^3 + \sqrt{(\sin x)}$ for $0 \leqslant x \leqslant \frac{\pi}{4}$.

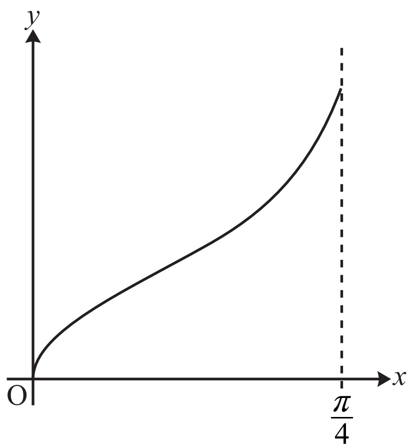

Fig. 3

(i) Use the trapezium rule with 4 strips to estimate the area of the region bounded by the curve, the x-axis and the line $x = \frac{\pi}{4}$, giving your answer to 3 decimal places. **[4]**

(ii) Suppose the number of strips in the trapezium rule is increased. Without doing further calculations, state, with a reason, whether the area estimate increases, decreases, or it is not possible to say. **[1]**

2 Fig. 2 shows the curve $y = \sqrt{1 + x^2}$.

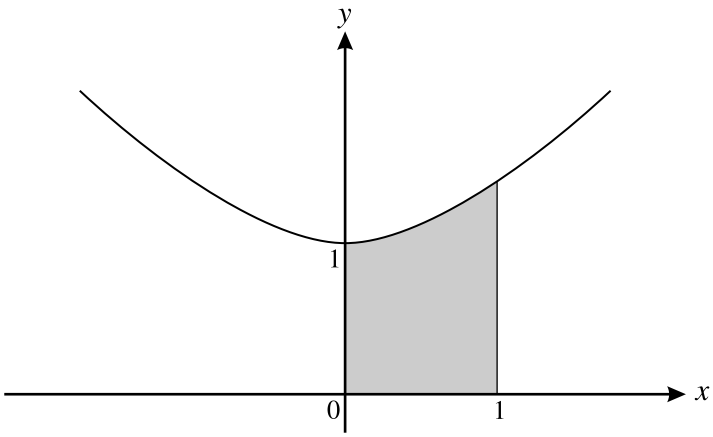

Fig. 2

(i) The following table gives some values of x and y.

x	0	0.25	0.5	0.75	1
y	1	1.0308		1.25	1.4142

Find the missing value of y, giving your answer correct to 4 decimal places.

Hence show that, using the trapezium rule with four strips, the shaded area is approximately 1.151 square units. **[3]**

(ii) Jenny uses a trapezium rule with 8 strips, and obtains a value of 1.158 square units. Explain why she must have made a mistake. **[2]**

(iii) The shaded area is rotated through $360°$ about the x-axis. Find the exact volume of the solid of revolution formed. **[3]**

3 Fig. 4 shows the curve $y = \sqrt{1 + e^{2x}}$, and the region between the curve, the x-axis, the y-axis and the line $x = 2$.

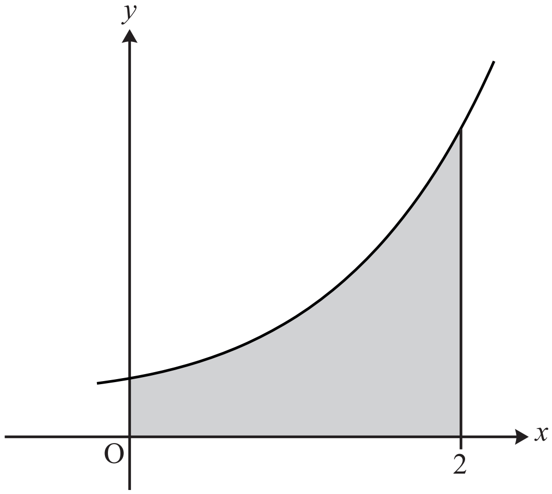

Fig. 4

(a) Find the exact volume of revolution when the shaded region is rotated through 360° about the x-axis. **[4]**

(b) **(i)** Complete the table of values, and use the trapezium rule with 4 strips to estimate the area of the shaded region. **[3]**

x	0	0.5	1	1.5	2
y		1.9283	2.8964	4.5919	

(ii) The trapezium rule for $\int_0^2 \sqrt{1 + e^{2x}}\,dx$ with 8 and 16 strips gives 6.797 and 6.823, although not necessarily in that order. Without doing the calculations, say which result is which, explaining your reasoning. **[1]**

4 **(i)** Complete the table of values for the curve $y = \sqrt{\cos x}$.

x	0	$\frac{\pi}{8}$	$\frac{\pi}{4}$	$\frac{3\pi}{8}$	$\frac{\pi}{2}$
y		0.9612	0.8409		

Hence use the trapezium rule with strip width $h = \frac{\pi}{8}$ to estimate the value of the integral $\int_0^{\frac{\pi}{2}} \sqrt{\cos x}\, dx$, giving your answer to 3 decimal places. **[3]**

Fig. 4 shows the curve $y = \sqrt{\cos x}$ for $0 \leqslant x \leqslant \frac{\pi}{2}$.

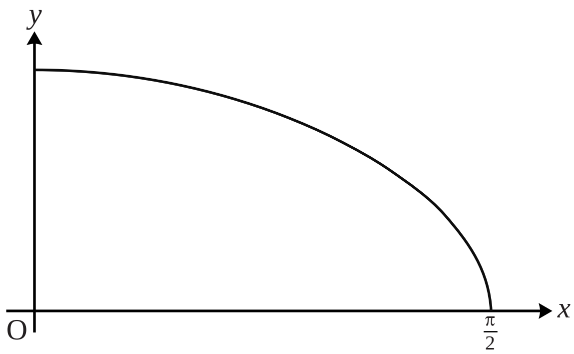

Fig. 4

(ii) State, with a reason, whether the trapezium rule with a strip width of $\frac{\pi}{16}$ would give a larger or smaller estimate of the integral. **[1]**

5 **(i)** Use the trapezium rule with four strips to estimate $\int_{-2}^{2} \sqrt{1 + e^x}\, dx$, showing your working. **[4]**

Fig. 1 shows a sketch of $y = \sqrt{1 + e^x}$.

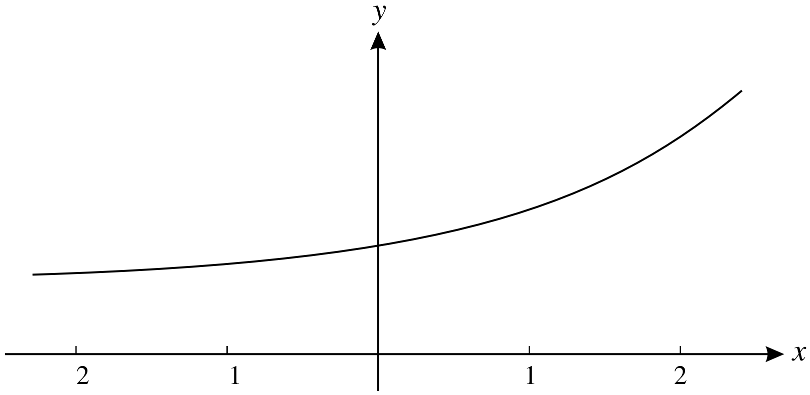

Fig. 1

(ii) Suppose that the trapezium rule is used with more strips than in part **(i)** to estimate $\int_{-2}^{2} \sqrt{1 + e^x}\, dx$. State, with a reason but no further calculation, whether this would give a larger or smaller estimate. **[2]**

6 Two students are trying to evaluate the integral $\int_1^2 \sqrt{1+\mathrm{e}^{-x}}\,\mathrm{d}x$.

Sarah uses the trapezium rule with 2 strips, and starts by constructing the following table.

x	1	1.5	2
$\sqrt{1+\mathrm{e}^{-x}}$	1.1696	1.1060	1.0655

(i) Complete the calculation, giving your answer to 3 significant figures. [2]

Anish uses a binomial approximation for $\sqrt{1+\mathrm{e}^{-x}}$ and then integrates this.

(ii) Show that, provided e^{-x} is suitably small, $\left(1+\mathrm{e}^{-x}\right)^{\frac{1}{2}} \approx 1+\frac{1}{2}\mathrm{e}^{-x} \quad \frac{1}{8}\mathrm{e}^{-2x}$. [3]

(iii) Use this result to evaluate $\int_1^2 \sqrt{1+\mathrm{e}^{-x}}\,\mathrm{d}x$ approximately, giving your answer to 3 significant figures. [3]